Table 1 Parameters for two rocket-assisted projectiles

	155 mm M549	8-in. M650
Burn-time, s	2.68	2.95
$\dot{m}$, kg/s	−1.12	−1.86
$\dot{I}_x$, kg-m²/s	−0.00285	−0.00714
$\dot{I}_t$, kg-m²/s	−0.0379	−0.0803
R_n, m	0.0222	0.0317
X_n, m	0.326	0.365
V, m/s	481	603
C_{l_p}	−0.011	−0.011
$C_{M_q}+C_{M_{\dot\alpha}}$	−13.2	−11.9
$(C_{J_p})_{max}$	0.019	0.012
C_{J_q}	−0.61	−0.34

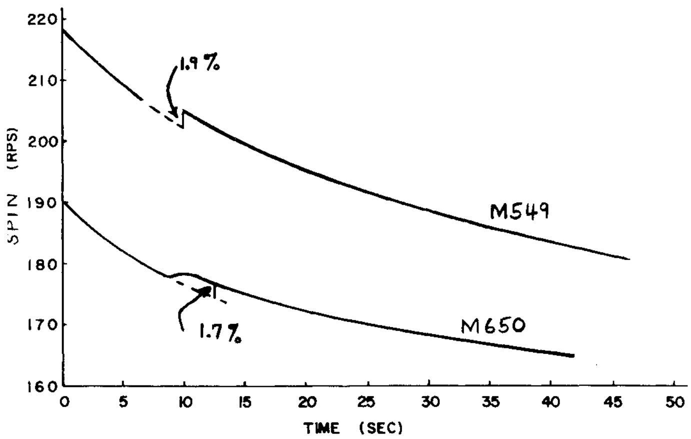

Fig. 1 Measured spin histories for two rocket-assisted projectiles showing the percentage spinup during burning.

that the aerodynamic moments are nondimensionalized[3]:

$$J_p = \tfrac{1}{2}\rho V S l^2 C_{J_p} \qquad J_q = \tfrac{1}{2}\rho V S l^2 C_{J_q} \tag{11}$$

In Table 1, the average values of $\dot{I}_x, \dot{I}_t, \dot{m}$ and other appropriate parameters are given for two rocket-assisted projectiles, the 155 mm M549 and the 8-in. M650. According to this table, the pitch jet damping coefficient is much smaller than the aerodynamic pitch damping coefficient and can be neglected, but the spin jet damping coefficient is the same size as the aerodynamic spin damping coefficient. We would therefore expect the spin to be affected by this term during burning.

Another way to estimate the effect of the spin jet damping is to consider Eq. (8) for no aerodynamic moment:

$$\dot{p}/p = -\epsilon\gamma(\dot{I}_x/I_x) \tag{12}$$

where

$$\gamma = -(J_p)_{max}/\dot{I}_x$$

γ can be computed from the table and is 0.90 and 0.87 for the M549 and M650, respectively. Thus the percentage change in spin during burning can be approximately related to the percentage change in the spin moment of inertia:

$$\frac{\Delta p}{p} = -\epsilon\gamma\left(\frac{\Delta I_x}{I_x}\right) = \begin{cases} 0.046\epsilon & \text{(M549)} \\ 0.033\epsilon & \text{(M650)} \end{cases} \tag{13}$$

Experimental Results

Recently yawsonde data have been obtained[4,5] for the spin during burning of both the M549 and the M650. Sample spin histories for each RAP are given in Fig. 1. The spin curve before burning was extended through burning and the percentage change in spin during burning was determined. It was 1.9% for the M549 and 1.7% for the M650. These values correspond to ϵ values of 0.41 and 0.52, respectively.

Conclusions

1) A theoretical model for spin jet damping has been derived which predicts a percentage increase in spin proportional to the percentage decrease in the spin moment of inertia.

2) Experimental results show approximately 2% increase in spin, which is about half the predicted maximum increase.

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Inversion of a Class of Complex Matrices

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Introduction

INVERSION of complex matrices is required for the solution of many problems. One approach to the inversion problem is simply to use complex arithmetic with a standard inversion algorithm. Another approach is to use various identities to transform the problem into one which requires only real arithmetic. The latter approach is usually not as efficient as the first with respect to either speed or storage requirements; and sometimes requires certain nonsingularities which may not be guaranteed a priori.[1-4]

This Note is concerned with the inversion of a certain class of complex matrices which frequently arise in practice, namely, nonsingular matrices for which each column (or row) is either real or the complex conjugate of another column (or row). Probably the most common application is in the diagonalization of a real matrix A by a similarity transformation $T^{-1}AT$ where T is the modal matrix constructed from the right eigenvectors of A. The inverse of T is the transpose of the matrix of left eigenvectors of A, so the similarity transformation also provides the left eigenvectors as a byproduct.

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It is shown that for this class of complex matrices, the inverse can always be computed from the inverse of a single real matrix of the same order as the complex matrix. The results can thus be used to significantly reduce the digital computer memory and processing time requirements for applications to matrices of large order.

Formulation

Let A be an $n \times n$ nonsingular complex matrix such that each column of A is either real or the complex conjugate of another column. The real and imaginary parts of A are denoted by B and C, respectively, so that $A = B + jC$, where $j = \sqrt{-1}$. The columns of A, B, and C are denoted by A_i, B_i, and C_i $(i = 1,2,\ldots,n)$. For development purposes, the first k columns of A are assumed to be conjugate complex pairs and the last $n-k$ columns are assumed to be real, but this structure is not required for implementation of the results.

The primary problem to be addressed is the computation of the inverse A^{-1} of A using real arithmetic and a single real matrix inversion. A secondary problem is the extension to a matrix $\hat{A}$ which has row characteristics analogous to the column characteristics of A.

Solution

From the above definitions

$$\begin{aligned} A &= [A_1 A_2 \ldots A_n] \\ &= [A_1 A_1^* A_3 A_3^* \ldots A_{k-1} A_{k-1}^* A_{k+1} \ldots A_n] \end{aligned} \tag{1}$$

where the asterisk denotes the complex conjugate. Also,

$$A_i = B_i + jC_i \qquad A_i^* = B_i - jC_i \tag{2}$$

from which it is readily shown that

$$B_i = A_i/2 + A_i^*/2 \qquad C_i = -jA_i/2 + jA_i^*/2 \tag{3}$$

Now, let the real matrix D be defined as

$$D = [B_1 C_1 B_3 C_3 \ldots B_{k-1} C_{k-1} B_{k+1} \ldots B_n] \tag{4}$$

From Eqs. (1), (3), and (4),

$$D = AK \tag{5}$$

where K is the $n \times n$ block diagonal matrix

$$K = \begin{bmatrix} J & & & & & \\ & J & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & J & \\ & & & & & & I \end{bmatrix} \tag{6}$$

with $k/2$ blocks of

$$J = \begin{bmatrix} 1/2 & -j/2 \\ 1/2 & j/2 \end{bmatrix} \tag{7}$$

and I is an $(n-k) \times (n-k)$ identity matrix. Clearly, the determinant $|K|$ of K is

$$|K| = |J|^{k/2} = (j/2)^{k/2} \tag{8}$$

so that K is nonsingular. Since A is also nonsingular by definition, it follows that D is nonsingular. Premultiplication of Eq. (5) by A^{-1} and postmultiplication by D^{-1} gives

$$A^{-1} = KD^{-1} \tag{9}$$

Now, let the rows of A^{-1} and D^{-1} be denoted by a_i and d_i $(i = 1,2,\ldots,n)$, respectively. From Eqs. (6), (7), and (9),

$$\begin{aligned} &a_i = d_i/2 - jd_{i+1}/2 \qquad a_{i+1} = a_i^* \qquad (i = 1,3,\ldots,k-1) \\ &a_i = d_i \qquad (i = k+1; k+2,\ldots,n) \end{aligned} \tag{10}$$

Equations (10) define the real and imaginary parts of the rows of the inverse of the complex matrix A in terms of the rows of the inverse of the real matrix D given by Eq. (4). Thus, A^{-1} can be constructed directly from D^{-1}, which can be computed from any suitable real matrix inversion algorithm.

Implementation of these results does not require the columns of A to be ordered as described by Eq. (1), but it is necessary to know which columns of A are conjugate complex pairs. The corresponding columns of D can then be constructed from the real and imaginary portions of the first column of each pair. The row correspondence between A^{-1} and D^{-1} is then the same as the columns of A and D.

Extension of this general result to a matrix $\hat{A}$ of the form described earlier is straightforward. Upon defining $A = \hat{A}^T$, where the superscript T denotes the transpose, and using the relation $\hat{A}^{-1} = [(\hat{A}^T)^{-1}]^T$, it follows that $\hat{A}^{-1} = (A^{-1})^T$. Thus, the rows of D^{-1} are related to the columns of $\hat{A}^{-1}$ ina manner analogous to Eqs. (10). But, upon defining $\hat{D} = D^T$, it follows that $\hat{D}^{-1} = (D^{-1})^T$. It is now observed that the relationship between the rows of $\hat{D}$ and $\hat{A}$ is the same as the relationship between the columns of D and A. Furthermore, the relationship between the columns of $\hat{D}^{-1}$ and $\hat{A}^{-1}$ is the same as the relationship between the rows of D^{-1} and A^{-1}.

Computer Requirements Savings

A significant reduction in computer memory and processing time requirements can be realized by implementation of the above results.

Assuming a priori knowledge of the structure of the complex matrix to be inverted, the complete complex matrix is not required. In fact, all the information required to define the complex matrix, except perhaps for some bookkeeping data, can be stored in a single real matrix of the same order as the complex matrix. The primary data required is the real part of each real column (or row), and the real and imaginary parts of one column (or row) of each conjugate complex pair. The same requirements hold for the inverse of the matrix. An overall savings in memory requirements on the order of 50% can thus be expected.

Each complex arithmetic multiplication requires four real arithmetic multiplications and two real arithmetic additions. Assuming multiplications are the primary factor in the processing time required to invert a matrix, the elimination of the complex arithmetic requirement can be expected to reduce the processing time requirements on the order of 75%.

Conclusions

An efficient method of computing the inverse of a nonsingular complex matrix having columns (or rows) which are real or occur in conjugate complex pairs has been presented. The method requires only a single inversion of a real matrix of the same order as the complex matrix. Savings in computer memory/processing time requirements on the order of 50-75% can be expected because of the elimination of the need for complex numbers and complex arithmetic.

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